Two-Stream Instability Model With Electrons Trapped in Quadrupoles

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Abstract

We formulate the theory of the two-stream instability (e-cloud instability) with electrons trapped in quadrupole magnets. We show that a linear instability theory can be sensibly formulated and analyzed. The growth rates are considerably smaller than the linear growth rates for the two-stream instability in drift spaces and are close to those actually observed.

1 Introduction

The Proton Storage Ring (PSR) at Los Alamos has been troubled for some time, [1], [2], [3], [4], [5], [6], [7], [8], [9], by an instability that is probably a two-stream instability of the proton beam with background electrons, i.e. an electron-cloud instability. We have previously considered the possibility that the instability for a bunched beam occurred because of electrons in drift regions that were renewed from turn to turn, [10]; in this case the phase memory of the coherent motion has to reside in the proton beam which excited the fresh electrons on each turn which then drove the tail of the proton bunch to larger amplitudes. In this note we will consider instead the possibility that the instability is due to electrons that survive from turn to turn. The most likely place in the ring where the electrons can survive with coherent phase information from turn to turn is in the quadrupoles, where they are trapped in the magnetic mirrors formed by the cusp-shaped fields and can drive the e-p instability in a similar way to free electrons. In this note we will present a simple model of this two-stream instability with electrons trapped in quadrupoles.

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1.1 Electron Trapping and Dynamics

A major assumption of this note is that there are abundant electrons in the PSR; experimentally this has been observed, though the source is not completely clear. It is likely that some form of beam induced multipactor gives rise to the electrons, perhaps initiated by a very small number of lost beam particles, though other explanations are possible. Normally, one would expect that with a bunched beam electrons would be expelled during the beam gap and that one could not have an *e-p* instability; however, the electrons, however they are produced, cannot be driven quickly to the walls in the quadrupoles which act in the transverse direction as very effective magnetic mirrors. It thus seems possible that electrons in the quadrupoles could drive the *e-p* instability. To investigate this possibility further in this section we will make simple estimates of the electron motion in quadrupoles to establish that electrons can be trapped there for multiple turns and thus carry coherent phase information to drive the instabillity. A more accurate investigation of the electron motion in the complex geometry can and should be done using computer codes, [11].

The dominant aspect of electron motion in the quads is the rapid rotation about the magnetic field lines; the cyclotron frequency is

$$f_c = \frac{eB}{2\pi mc}, \tag{1}$$

where e is the charge, B is the magnetic field, m is the mass, and c is the speed of light. For electrons we have

$$f_c = 2.8\mathcal{B} \text{ GHz}, \tag{2}$$

where $\mathcal{B}$ is the magnetic field measured in kilogauss. Thus, even very low fields near the axis give rise to cyclotron frequencies that are hundreds of MHz; most electrons will have cyclotron frequencies that are multiple GHz. The radius of this rotational motion is, for electrons,

$$\rho = 3.37 * 10^{-3} \frac{\sqrt{\mathcal{E}}}{\mathcal{B}} \text{ cm}, \tag{3}$$

where $\mathcal{E}$ is the transverse electron energy in eV. Only very energetic electrons in low field regions will have gyroradii approaching 1 cm; most will have gyroradii that are much less than 1 mm. Electrons are thus confined transversely to the magnetic field on cyclotron orbits of small radii and many are confined longitudinally (for electrons) along the magnetic field by the increasing magnetic field with radius, i.e. by 'mirror' confinement. (Note that longitudinal for the electrons is transverse to the beam direction.)

Of course, particles with large components of velocity parallel to the magnetic field, i.e. those in the 'loss cone', are not confined; presumably these give rise to the electron 'tracking' that has been observed in the quadrupoles. We will ignore the rapid electron cyclotron motion in the quads and concentrate on the longitudinal electron mirror motion and transverse drifts due to electric

fields and to magnetic field non-uniformity, i.e. a 'guiding center' description of the trapped electrons.

In the transverse direction (for electrons) there are three components of electron drift, that due to the gradient in the magnetic field, the so-called ∇B drift, that due to the field line curvature, and that due to any electric fields that are present, the $E \times B$ drift. These drifts give rise to electron velocities perpendicular to the magnetic field; in fact, in the quads, the drifts are along the direction of the beam axis and thus can lead to electron loss out the ends of the quads.

The ∇B drift and curvature velocities are given by

$$V_{\nabla B} = \frac{m(v_\perp^2 + 2v_\parallel^2)}{2eB} \frac{\hat{b} \times \nabla B}{B} c, \tag{4}$$

where $\hat{b}$ is a unit vector in the direction of the magnetic field, the $v_\perp^2$ term is due to the gradient drift and the $v_\parallel^2$ term is due to the curvature. If we assume the parallel and perpendicular electron velocities to be roughly the same and adopt the usual model of quadrupole magnetic fields in which a component is linear in transverse displacement from the axis, i.e.

$$B = B'r, \tag{5}$$

then, defining the ∇B confinement time, $T_{\nabla B}$ to be the time for an electron to drift half the length of a quad, L_Q, we get

$$T_{\nabla B} = \frac{eB'r^2 L_Q}{4Ec}, \tag{6}$$

where E is the thermal energy of the electron. This becomes

$$T_{\nabla B} = \frac{\bar{B}'\bar{r}^2 \bar{L}_Q}{4\mathcal{E}} \ \mu\text{sec}, \tag{7}$$

where $\bar{B}'$ is the field gradient in T/m, $\bar{r}$ is the radius in cm, $\mathcal{E}$ is the energy in eV, and $\bar{L}_Q$ is the quad length in cm. As an example typical of the PSR, if we take $\bar{B}' = 3.7$, $\bar{r} = 2.5$, and $\bar{L}_Q = 47$, then

$$T_{\nabla B} = \frac{272}{\mathcal{E}} \ \mu\text{sec}. \tag{8}$$

Note that this is an overestimate of the drifts since the actual drift reverses sign as the electrons move out along the magnetic field lines toward the poles. If the electrons only have energies that are a few hundred eV then the confinement time is tens to hundreds of turns and is probably longer than the growth time for the *e*-*p* instability.

The $E \times B$ drift velocity is given by

$$V_{E \times B} = \frac{E_\perp \times B}{|B|^2} c \tag{9}$$

The electric field is due to the proton beam and to any electrons that are present. The electric potential due to the proton beam alone is given by

$$e\phi = \frac{2eI}{\beta c}, \tag{10}$$

where β is the beam velocity scaled by the speed of light and I is the (time-dependent) beam current. The beam current varies by 100% in one revolution period (the beam is bunched), but we will estimate drifts using the average current and resulting field. Note that electrons spend a lot of time near the magnetic mirror points where we expect that the E field will mostly be parallel to the B field and will give rise to only small drifts. Nevertheless, the $E \times B$ drift velocity due to this term alone, assuming it acts all the time, would give an electron confinement time of

$$T_{E\times B} = 8.34 \frac{\beta \bar{B}' \bar{r}^2 \bar{L}_Q}{\mathcal{I}} \text{ nsec}, \tag{11}$$

where the current, $\mathcal{I}$, is measured in amps. if we again take $\bar{B}' = 3.7$, $\bar{r} = 2.5$, $\mathcal{I} = 10$, and $\bar{L}_Q = 47$, then $T_{E\times B} = 761$ nsec, i.e. electrons would be confined for several turns, even with this overestimate of the $E \times B$ drift. With a more realistic calculation, including the full orbit dynamics of the electrons and the reverse drifts that occur when only the electrons are present, it is likely that the electrons will be confined for many turns.

Electrons to the left and right of the beam, horizontally, are free to move vertically (initially) until they move out radially along the field line to a region of greater field strength. Electrons above and below the beam, vertically, are free to move horizontally (initially) until they move out radially along a field line to a region of greater field strength. A complete model of the electron motion is very complicated, but a simple model will suffice to treat the motion of the center of mass of the electrons for oscillations near the beam axis. Let us note that for electrons that can move vertically, i.e. those to the left and right of the beam, the restoring mirror force exactly vanishes at zero vertical position and the restoring force reverses sign there. For electrons that can move horizontally, i.e. those above and below the beam, the restoring mirror force exactly vanishes at zero horizontal position and the restoring force reverses sign there. Thus, in both transverse directions we should expect the restoring potential for an electron to be approximately a harmonic oscillator potential near the axis. To see this in more detail, let us begin with the equation from Krall and Trivelpiece, [12], for the equation of motion along a field line of a particle in a magnetic field

$$\frac{d^2 s}{dt^2} \approx -\frac{v_{\perp 0}^2}{2B_0} \frac{\partial B}{\partial s}, \tag{12}$$

where s is the distance along the field line, $v_{\perp 0}$ is the initial value of the transverse velocity, and B_0 is the initial value of the magnitude of the magnetic field. The components of the quadrupole field are

$$B_x = B_0' y, \tag{13}$$

$$B_y = B_0' x. \tag{14}$$

We thus see that

$$\frac{\partial B}{\partial s} = \frac{2B_0' xy}{x^2 + y^2}. \tag{15}$$

From this we see that a particle that starts at $x = x_0$, $y = 0$ satisfies the approximate equation

$$\frac{d^2 y}{dt^2} \approx -(\frac{B_0' v_{\perp 0}^2}{B_0 x_0}) y, \tag{16}$$

i.e. it is approximately a harmonic oscillator with a squared angular frequency of

$$\omega_m^2 = \frac{B_0' v_{\perp 0}^2}{B_0 x_0}. \tag{17}$$

But $B_0 \approx B_0' x_0$, so

$$\omega_m^2 \approx \frac{v_{\perp 0}^2}{x_0^2}. \tag{18}$$

It thus appears that modeling the mirror trapping of the electrons by a harmonic oscillator potential, but with a large spread in oscillation frequencies, should be a fairly good approximation.

2 Dipole Model of the *e*-*p* Instability

In this section, in order to find thresholds and growth rates, we will present a simple theory of the *e*-*p* instability. The model for the linear theory of the instability in this section that we use is similar to the theory of Keil and Zotter, [13]. We model the proton beam by the beam centroid at each azimuthal position around the ring. The background electrons have a complex distribution both in physical and in velocity space determined by their formation, capture in the quadrupoles, interaction with the proton beam, and loss, as discussed in the previous section. We cannot hope to accurately model all of these effects in an analytic theory; we will simply assume that the electrons have a distribution in the squared magnetic bounce frequency, $g_m = \omega_m^2$, and that at each bounce frequency those electrons are described by their centroid position, with electrons at a different bounce frequency having a different centroid. We assume the proton beam moves at a constant azimuthal velocity around the ring and is subject to a constant transverse focusing force that produces betatron oscillations at the betatron frequency, i.e. we make the smooth approximation, [14]. We only model proton beam and electron motion in one transverse direction. The protons and electrons are assumed to interact with each other via a force

that is linear in the relative displacement of the centroids of the protons and electrons. The equations of motion for the centroids are thus given by

$$(\frac{\partial}{\partial t} + \omega_0 \frac{\partial}{\partial \theta})^2 y_p + \Gamma_d (\frac{\partial}{\partial t} + \omega_0 \frac{\partial}{\partial \theta}) y_p = -\omega_\beta^2 y_p + \omega_p^2 (Y_e - y_p) \tag{19}$$

$$\frac{\partial^2 y_{em}}{\partial t^2} + \omega_m^2 y_{em} = \omega_e^2 (y_p - y_{em}) \tag{20}$$

where $y_p(\theta, t)$ is the proton centroid position at an azimuth, θ, around the machine and time, t, ω_0 is the proton beam angular revolution frequency in the machine, and ω_β is the angular betatron frequency of the protons. The proton beam centroid only responds to the net electron centroid position, Y_e, which is given by

$$Y_e = \int f(g_m) y_{em}(\theta, t) dg_m, \tag{21}$$

where $f(g_m)$ is the equilibrium distribution function of electrons in the squared bounce frequency and $y_{em}(\theta, t)$ is the centroid of electrons with a particular bounce frequency. The coupling frequencies ω_p and ω_e are given by

$$\omega_e^2 = \frac{2 N_p r_e c^2}{\pi b (a+b) R} \tag{22}$$

$$\omega_p^2 = (\frac{F m_e}{\gamma m_p}) \omega_e^2 \tag{23}$$

with N_p the number of protons in the machine, r_e the classical electron radius, c the velocity of light, γ the relativistic factor of the proton beam, a and b the sizes of the proton beam, F the neutralization fraction of electrons, and R the effective radius of the ring. Note that the inter-species force is assumed to depend linearly on the distance between the beam centroids; this is approximately correct for small amplitudes of oscillation, but clearly fails at larger oscillation amplitudes.

Also note that we have inserted a linear damping term with coefficient Γ_d into the proton equation to account for the chromatic spread in proton revolution frequencies; the different revolution frequencies will give different longitudinal velocities which will Landau damp the transverse oscillations. A more extensive model would have the proton beam described by a distribution function in the azimuthal direction and take into account the Landau damping due to the spread in azimuthal velocities. The approximation we have adopted mimics this damping and has the same functional dependence as the result of this more extensive model (see below), i.e. the damping depends on 1) the energy spread, 2) the momentum compaction factor, and 3) the mode number (through the derivative in the damping term). Thus, this damping term will give rise to the correct qualitative behavior with the correct functional dependencies, i.e. damping of off-axis oscillations as they phase-mix away. We can estimate this damping rate of transverse oscillations due to this spread to be the chromatic

fractional tune spread times the betatron frequency. Note that the chromatic fractional tune spread is just the chromaticity times the energy spread, i.e. it measures the longitudinal velocity spread and its influence on the transverse oscillations. We do not include the transverse tune spread due to space charge and machine nonlinearities because we are using a dipole model and the centroid motion of the protons does not depend on these terms.

$$\Gamma_d \sim (\frac{\Delta\nu}{\nu})\frac{\omega_\beta}{2\pi}. \tag{24}$$

Because we are using an unbunched beam model, i.e. the smooth approximation, the average neutralization around the ring will be smaller than the neutralization in the quadrupoles by roughly the ratio of the ratio of total quadrupole length to the ring circumference; thus the neutralization fraction in a quadrupole will be about 20 times F since quadrupoles are about 10% of the circumference and only about half the electrons can move vertically.

We have seen in the context of the drift space instability model, [10], that bunching doesn't have a large effect on the instability, and we assume the same to be true here. There seems to be no simple way to incorporate bunching; a moderately realistic model would result in a dispersion equation which would be an infinite matrix eqation with all unbunched beam modes coupled. The unbunched beam model of this paper would then be just the diagonal approximation to this matrix equation. It is likely that an extensive numerical investigation would be required to resolve the behavior.

The above model is overly simplified, but contains most of the important physics. It will break down, of course, if the electron loss rate is too high. Of couse, we are also assuming that the background electron density, on average, is constant so that if electron generation and loss rates fluctuate rapidly our model should fail.

The model of Bosch, [15], for the effect of beam gaps on the trapped ion instability in an electron ring also considers the effect of a large spread of (ion) oscillation frequencies on the instability, and his formulation is similar to ours.

If we assume that the perturbations have a dependence on time and angle proportional to $e^{-i\omega t+in\theta}$, then the equations become

$$(-(\omega - n\omega_0)^2 - i\Gamma_d(\omega - n\omega_0) + \omega_\beta^2 + \omega_p^2)y_p = \omega_p^2 Y_e \tag{25}$$

$$(\omega_e^2 + \omega_m^2 - \omega^2)y_e = \omega_e^2 y_p. \tag{26}$$

Solving equation 26 for y_e and using equations 21 and 25 we find

$$((\omega - n\omega_0)^2 + i\Gamma_d(\omega - n\omega_0) - \omega_\beta^2 - \omega_p^2) = -\omega_e^2\omega_p^2 \int \frac{f(g_m)}{g_m + \omega_e^2 - \omega^2} dg_m, \tag{27}$$

where we have used the definition of $g_m = \omega_m^2$. We have to deal with the singularity in the integral on the right hand side of this equation. We adopt

the Landau prescription, see [12], where the integral is replaced by the principal value plus πi times the residue at the pole;

$$\int \frac{f(g_m)}{g_m + \omega_e^2 - \omega^2} dg_m = ⨍ \frac{f(g_m)}{g_m + \omega_e^2 - \omega^2} dg_m + \pi i f(\omega^2 - \omega_e^2). \qquad (28)$$

Equation 27 thus becomes

$$\begin{aligned} ((\omega - n\omega_0)^2 + i\Gamma_d(\omega - n\omega_0) - \omega_\beta^2 - \omega_p^2) \quad &= \quad -\omega_e^2\omega_p^2 ⨍ \frac{f(g_m)}{g_m + \omega_e^2 - \omega^2} dg_m \\ &\quad -\pi i \omega_e^2 \omega_p^2 f(\omega^2 - \omega_e^2), \qquad (29) \end{aligned}$$

where the bar through the integral sign indicates principal value. This is the dispersion relation for the two-stream mode. To solve it we have to specify the distribution function of electron bounce frequencies, f. Of course, there should be no electrons in the 'loss-cone', i.e. at zero ω_m^2, but otherwise the detailed distribution depends on their formation, capture in the quadrupoles, interaction with the proton beam, and loss. We will simply take one distribution as an example, one in which the distribution is constant between a minimum squared bounce frequency and a maximum squared bounce frequency; i.e.

$$\begin{aligned} f(g_m) \quad &= \quad \frac{1}{g_{max} - g_{min}} \quad g_{min} \leq g_m \leq g_{max}, \\ &\quad\; 0 \qquad\qquad\qquad \text{otherwise} \qquad (30) \end{aligned}$$

With this distribution the dispersion equation, 29, becomes

$$\begin{aligned} (\omega - n\omega_0)^2 \quad &= \quad \omega_\beta^2 + \omega_p^2 - i\Gamma_d(\omega - n\omega_0) \\ &\quad - \frac{\omega_e^2\omega_p^2}{g_{max} - g_{min}} (\ln(\frac{g_{max} - \omega^2 + \omega_e^2}{\omega^2 - \omega_e^2 - g_{min}}) + \pi i) \qquad (31) \end{aligned}$$

Though this is a transcendental equation and can't be solved exactly, we note that the coefficient of the logarithmic term is small and the logarithm varies slowly, so we can simply solve iteratively. The remainder of the equation is a quadratic for $\omega - n\omega_0$ and the resulting approximate solution is

$$\begin{aligned} \omega \quad \approx \quad & n\omega_0 - \frac{i\Gamma_d}{2} \\ & \pm \frac{1}{2} \big[4(\omega_\beta^2 + \omega_p^2) - \Gamma_d^2 \\ & - \frac{4\omega_e^2\omega_p^2}{g_{max} - g_{min}} (\ln(\frac{g_{max} - (n\omega_0)^2 + \omega_e^2}{(n\omega_0)^2 - \omega_e^2 - g_{min}}) + \pi i) \big]^{\frac{1}{2}} \qquad (32) \end{aligned}$$

We note the damping due to the phase mixing term, as expected, and the usual upper and lower sidebands. Note that we have taken $\omega \approx n\omega_0$ inside the

logarithm because the mode numbers are usually rather high $30-50$ and this is a good (few percent) approximation for the real part of the frequency. Let us expand just the imaginary term under the square root to find the damping and growth rates. For convenience define the real frequency shift to be

$$\omega_s^2 \equiv \frac{1}{4}\big[4(\omega_\beta^2+\omega_p^2)-\Gamma_d^2-\frac{4\omega_e^2\omega_p^2}{g_{max}-g_{min}}(\ln(\frac{g_{max}-(n\omega_0)^2+\omega_e^2}{(n\omega_0)^2-\omega_e^2-g_{min}}))\big] \qquad (33)$$

Note that to a good approximation $\omega_s \approx \omega_\beta$. Expanding the imaginary term in the square root we get

$$\omega \approx n\omega_0 - \frac{i\Gamma_d}{2} \pm \omega_s(1-\frac{\pi i\omega_e^2\omega_p^2}{2\omega_s^2(g_{max}-g_{min})}) \qquad (34)$$

Note that the upper side band (plus sign) is always damped, but that the lower side band can be unstable if

$$\frac{\pi\omega_e^2\omega_p^2}{\omega_s(g_{max}-g_{min})} > \Gamma_d, \qquad (35)$$

with growth rate given by

$$\gamma_{\mathrm{growth}} = \frac{\pi\omega_e^2\omega_p^2}{2\omega_s(g_{max}-g_{min})} - \frac{\Gamma_d}{2}. \qquad (36)$$

A number of modes in lower side bands can be unstable, limited only by the condition $\omega_e^2+g_{min}<\omega^2<\omega_e^2+g_{max}$, with roughly equal growth rates (there is some weak dependence on mode number in ω_s) and this is consistent with experiments where multiple modes are usually observed, [16].

2.1 Example

Let us look at an example typical of the PSR; let us take

$$a = b = 1.8 \text{ cm},$$

$$\omega_0 = 2\pi * 2.8 \text{ MHz},$$

$$\omega_\beta = 2.2 * \omega_0,$$

$$2\pi R = 89 \text{ m}$$

If we express the number of particles in the ring as

$$N_p = \mathcal{N} \times 10^{13}, \qquad (37)$$

then we can compute

$$\omega_e^2 = 1.76 * \mathcal{N} * 10^{17} \text{ sec}^{-2}, \qquad (38)$$

and

$$\omega_p^2 = 0.5182 * F * \mathcal{N} * 10^{14}\ \mathrm{sec}^{-2}. \tag{39}$$

In the PSR the measured vertical chromaticity is about -1.68 and the energy spread (typical conditions) is about 0.5% so we take the chromatic tune spread to be about 0.009, i.e. a fractional tune spread of 0.43%, then

$$\Gamma_d \approx 0.0252 * 10^6\ \mathrm{sec}^{-1}. \tag{40}$$

We take the frequency shift to be

$$\omega_s \approx \omega_\beta = 3.9 * 10^7\ \mathrm{sec}^{-1}. \tag{41}$$

To estimate g_{max} we use equation 18, setting the maximum transverse energy to the beam potential; the result is

$$g_{max} \approx 3.48 * \mathcal{N} * 10^{17}\ \mathrm{sec}^{-2}, \tag{42}$$

where we used $x_0 \approx a = 1.8$ cm. Note that we simply ignore g_{min}, i.e. assume it is zero; it only modifies our results by a small factor.

If we evaluate the threshold condition, equation 35, using equations 38, 39, 40, 41, and 42 we find the criterion for instability to be

$$F * \mathcal{N} > 0.188; \tag{43}$$

in other words, once the product of the particle number (times 10^{13}) and percent neutralization is about 19.0, we can expect instability. Recall that the neutralization fraction in quadrupoles will be about 20 times higher than F since quadrupoles are only about 10% of the ring and only half the electrons can move vertically. At threshold the growth time is infinite, but if, for simplicity, we assume that we are a factor of 2 above the threshold, $F * \mathcal{N} = 9.4 * 10^{-2}$, then using 38, 39, 40, 41, and 42 in equation 36 we find

$$\gamma_{\mathrm{growth}} \approx 12.6\ \mathrm{KHz}, \tag{44}$$

i.e. a growth time of about 222 turns. These estimates are only intended to show that the results seem to be within a factor of two or three of the observations and that the theory is thus a possible explanation of the observed instability.

3 Discussion

Our results show that electrons trapped in quadrupoles are a plausible explanation of the two-stream instability observed in the PSR. The growth times found are considerably closer to the observed values than the linear growth times derived from the instability treatment for electrons in drift spaces, [10]. The reason for this is that the electrons confined in quadrupoles have a very large frequency spread due to the wide variation in magnetic bounce frequencies as compared

to those in drift spaces which have only a very small spread in space charge confinement bounce frequencies. Thus, many fewer electrons are resonant at a particular frequency.

In addition, if the instability is due to electrons trapped in quadrupoles, then the transverse momentum kick given to the protons is easily explained; the momentum is transferred from the quadrupoles via the electrons, rather than having to be transferred only from electrons, as in the drift space theory.

Clearly a great deal more work can be done to refine this model. A kinetic description of the proton beam could be used, and would give a more sensitive dependence of the phase-mixing damping that depends on the detailed proton distribution. An investigation of different electron distribution functions, perhaps motivated by detailed simulation of electron formation and capture dynamics, would give threshold and growth rate estimates that are better founded than those in this note. The formulation of a bunched beam model would be considerably more difficult, but might be worthwhile. Finally, a composite model with both drift space electrons and quadrupole trapped electrons would be very difficult to analyze but might be necessary to fit all observations in real machines.

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